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- Simultaneous equations
- Compound interest
- Cumulative frequency
- Trigonometry (SOH CAH TOA)

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- Circle theorems (angles within a circle)
- Using the quadratic equation
- Probability
- Sine and Cosine rule
- Histograms

Simultaneous Equations: Key facts

$$4x - 2y = 7$$
$\times 3$

$$3x - 4y = 4$$
$\times 4$

Multiply the other equation by the amount of x in the other equation

$\times 3$ $12x - 6y = 21$

$\times 4$ $12x - 16y = 16$

Now Subtract the equations if the signs are the same or add them if it is different

$-6 \oplus 16 = 10y \rightarrow 10y = 5$

$21 - 16 = 5$ $y = \frac{5}{10}$, $y = 0.5$

$4x - 2 \times 0.5 = 7$

$4x - 1 = 7$, $4x = 8$

$x = \frac{8}{4}$ $x = 2$

Now substitute back in to an original equation

Questions to try:

Solve the simultaneous equations

$$3x + 7y = 26$$
$$4x + 5y = 13$$

Solve the simultaneous equations.

$$5a + 3b = 9$$
$$2a - 3b = 12$$

Compound interest: Key facts

Compound Interest

Worked Example 1   Long Method

£2000 is invested at 6% compound interest for 3 years.
Find: (a) the amount in the account at the end of the period.
and   (b) the interest accrued.

Amount after 1 year = 2000 + 6% of 2000 = 2000 + 120 = £2120

Amount after 2 years = 2120 + 6% of 2120 = 2120 + 127.20 = £2247.20

Amount after 3 years = 2247.20 + 6% of 2247.20 = 2247.20 + 134.83 = £2382.03

Interest accrued = £2382.03 - £2000 = £382.03

Compound Interest

Worked Example 1   Efficient Method

£2000 is invested at 6% compound interest for 3 years.
Find: (a) the amount in the account at the end of the period.

(a) Money at end of 3 years = 2000×1.06^3 = £2382.03 ✓

Explanation of the Method   Remember that 6% means $\dfrac{6}{100} = 0.06$

At the end of each year the money grows to 106% of its value at the start of the year = $\dfrac{106}{100} = 1.06$

After 1 year the money has been multiplied by 1.06 → 2000×1.06

After 2 years the money is again multiplied by 1.06 → $(2000 \times 1.06) \times 1.06$

After 3 years the money is again multiplied by 1.06 → $(2000 \times 1.06 \times 1.06) \times 1.06$

So after 3 years the money will have grown to £2000×1.06^3.

If the term had been 7 years and the interest rate 8% then we would simply have calculated 2000×1.08^7.

Remember that the amount is decreasing in will reduce every year. Per annum means every year. Depreciate means reducing.

Worked Example 1:

£2000 is invested at 6% compound interest for 3 years.

Find: (a) the amount in the account at the end of the period and (b) the interest accrued.

Q1. £600 is invested at 5% compound interest for 3 years.
Find: (a) the amount in the account at the end of the period. and (b)
the interest accrued.

**Q2.** £5000 is invested at 8% compound interest for 4 years.
Find: (a) the amount in the account at the end of the period (nearest £) and (b) the
interest accrued (nearest £)

Remember with the question below the decimal multiplier will be 100%-8%........

A car is bought for £17,000, a nice Toyota Celica.

The car loses 8% of its value every year due to old
age and wear and tear (depreciation).

How much will the car be worth after 3 years?
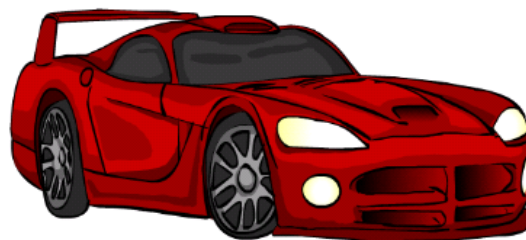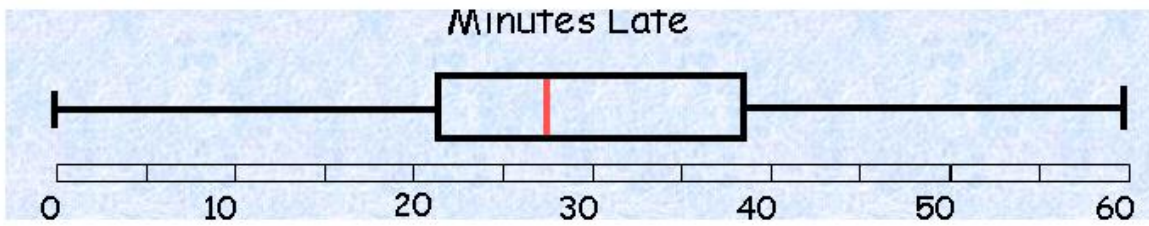
Cumulative Frequency: Key points

Cumulative frequency table

Minutes Late	Frequency	Upper Limit	Cumulative Frequency
0 - 10	5	< 10	5
10 - 20	8	< 20	13
20 - 30	22	< 30	35
30 - 40	12	< 40	47
40 - 50	8	< 50	55
50 - 60	5	< 60	60

Cumulative frequency just means running total.

Plot the upper limit!

Plotting the curve

IQR = 38 – 21
= 17 mins

LQ = 21
Median = 27
UQ = 38

Cumulative Frequency

Minutes Late

Minutes Late
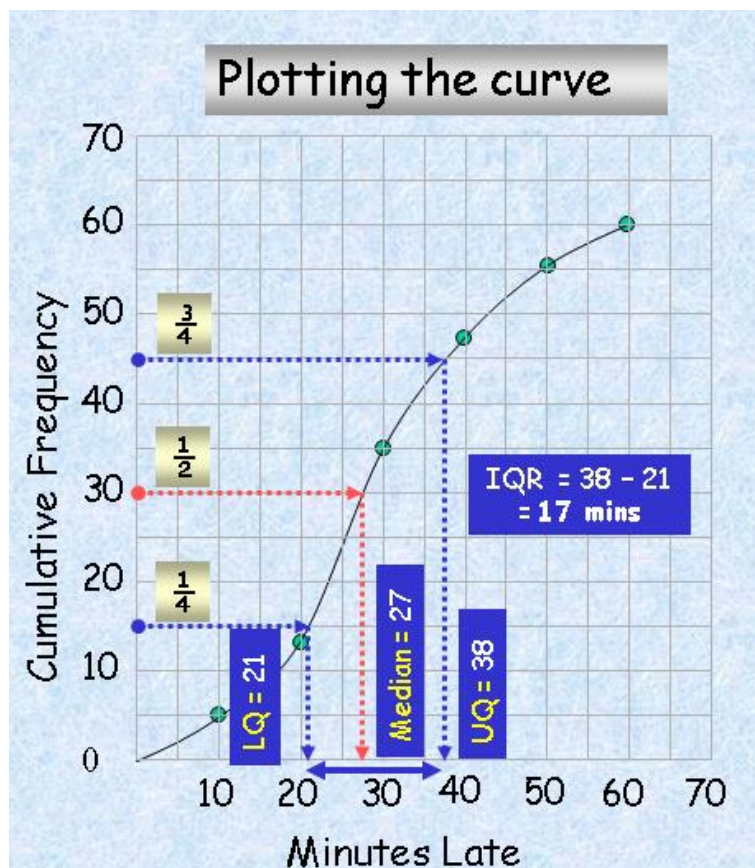
Lowest score	Lower Quartile	median	Upper quartile	Highest score

Question for you to try:

Time in seconds	Frequency	Cumulative frequency
$30 < t \le 35$	9	
$35 < t \le 40$	12	
$40 < t \le 45$	24	
$45 < t \le 50$	28	
$50 < t \le 55$	16	
$55 < t \le 60$	11	
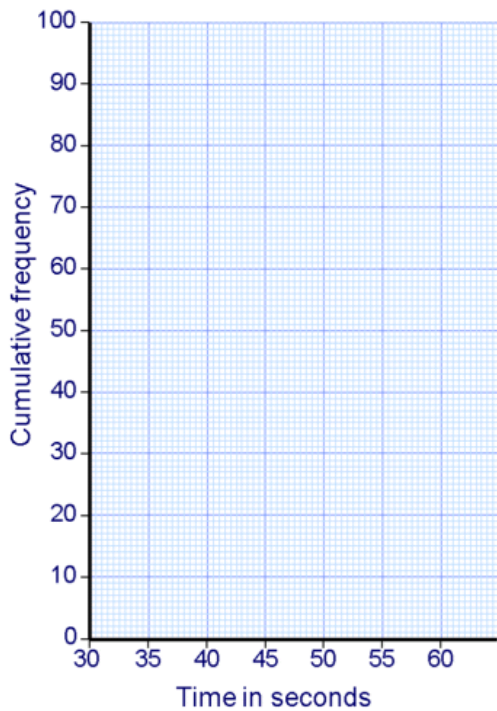
Trigonometry Key points:

S O H C A H T O A

$$Sine\ A = \frac{Opposite}{Hypotenuse}$$ $$SinA = \frac{O}{H}$$

$$Cosine\ A = \frac{Adjacent}{Hypotenuse}$$ $$CosA = \frac{A}{H}$$

$$Tangent\ A = \frac{Opposite}{Adjacent}$$ $$TanA = \frac{O}{A}$$

Convention for naming sides.
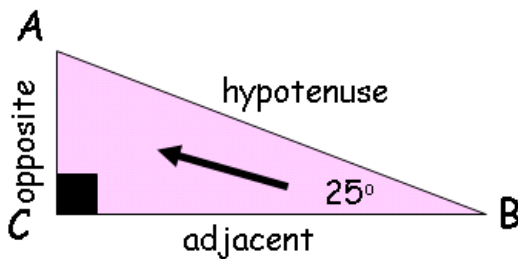
• The side opposite the right-angle is called the hypotenuse

• The side opposite a given angle is called the opposite side.

• The sides next to (or adjacent to) a given angle is called adjacent side.

A

Copposite
C

hypotenuse

25°

adjacent

B

The three stage process to find missing sides.

SOHCAHTOA

Step 1 - Which sides do we know?

Opp & Hyp   Adj & Hyp   Opp & Adj

O

A

x

13

40°

H
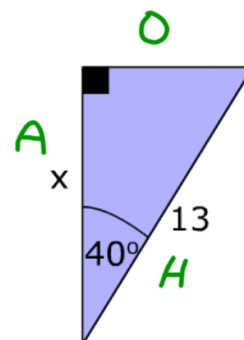
$$Cos\ A = \frac{A}{H}$$ $$Cos 40 = \frac{x}{13}$$ $$13 \times cos\ 40 = x$$

Now put the information into your calculator!

E.g.2
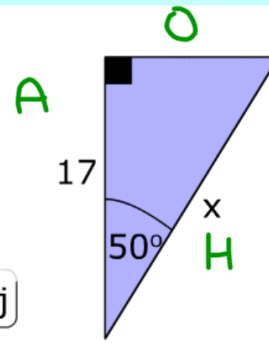
This is the bit no one likes;

SOHCAHTOA

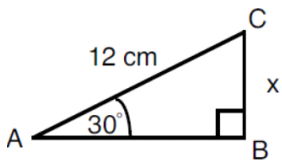Step 1 - Which sides do we know?

| Opp & Hyp | Adj & Hyp | Opp & Adj |

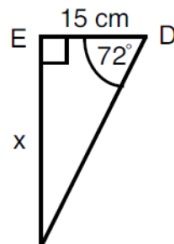$$Cos A = \frac{A}{H} \qquad Cos 50 = \frac{17}{x} \qquad x = \frac{17}{Cos 50}$$
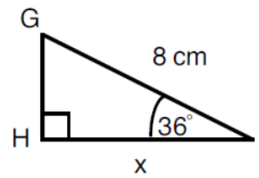
Some for you to try:

1).

12 cm, 30°, x, A, B, C

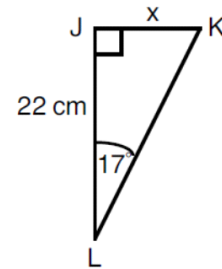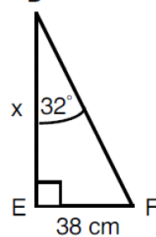2).

15 cm, 72°, E, D, x

3).

8 cm, 36°, G, H, x, I

4).

J, x, K, 22 cm, 17°, L

5).

x, 36°, 18 cm, A, B, C

6).

x, 32°, E, 38 cm, F

7).

G, 22 cm, 18°, H, x, I

8).

J, K, 22 cm, 17°, x, L
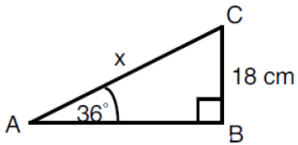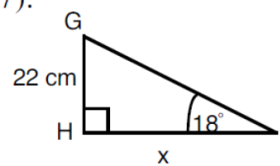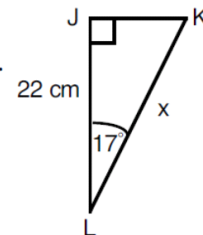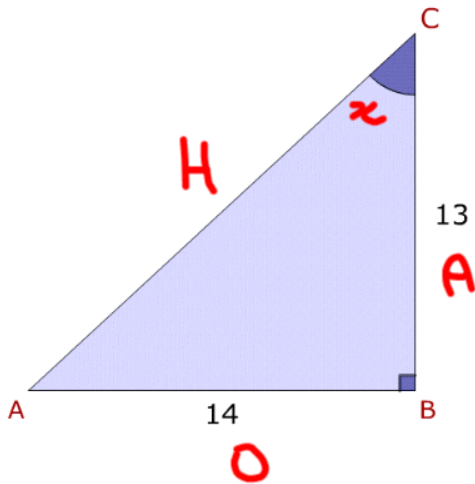
Finding missing angles using trigonometry:

Remember to use the inverse (-1)

SOH CAH TOA

$$tan = \frac{O}{A}$$

$$tan^{-1} \frac{14}{13}$$

Find the angles marked x.

1).

28 cm, C, 16 cm, A, x, B

2).

D, 3 cm, x, 6 cm, E, F

3).

G, 12 cm, 14 cm, H, x, I

4).

J, 7 cm, K, 12 cm, x, L

5).

U, 55 cm, T, x, S, 28 cm

6).

34 cm, P, x, Q, 46 cm, R

7).

C, 22 cm, x, 18 cm, B, A

8).

Z, 13 cm, 15 cm, V, x, W, L

9).

W, 57 cm, V, x, 39 cm, Z

10).

D, 13 cm, E, x, 12 cm

11).

P, x, 35 cm, Q, 25 cm, R

12).

M, 8 cm, N, x, 12 cm, O, R
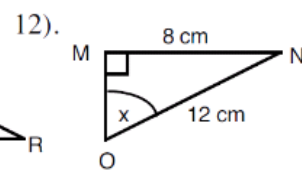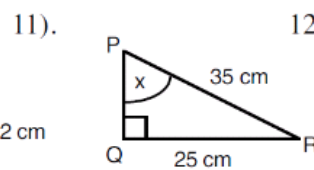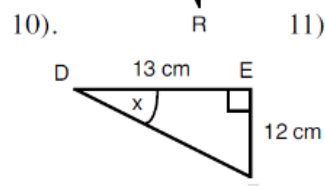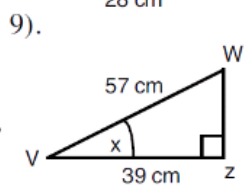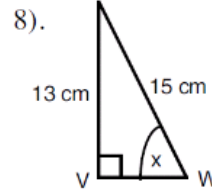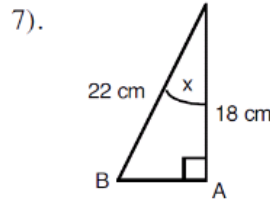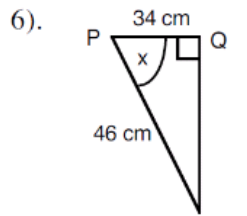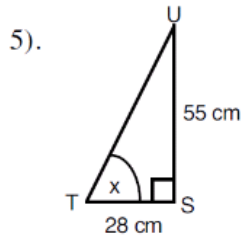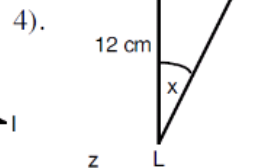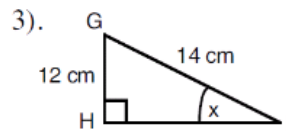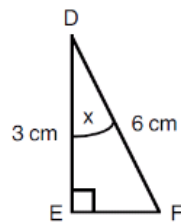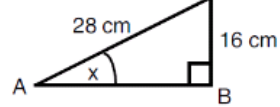
Circle theorems: You need to remember these:

Angles in Circles

SEMI-CIRCLE

AN ANGLE IN A SEMI-CIRCLE IS A RIGHT ANGLE

DIAMETER

GOES THROUGH THE CENTRE

Cyclic quadrilateral

This is a quadrilateral whose corners (vertices) all lie on the same circle

Opposite angles of a cyclic quadrilateral add up to 180°

a + b = 180° and

c + d = 180°

Angles in Segments

ANGLES IN THE SAME SEGMENT ARE EQUAL

Tangent and Radius

A radius drawn to the tangent is at right angles to that tangent

Mr Cottons favourite!

Two Tangent Theorem.

From any point outside a circle only two tangents can be drawn and they are equal in length.

PT = PQ

PT = PQ

Angles at the centre and the circumference (arrowhead)

×2

30°

÷2

60°
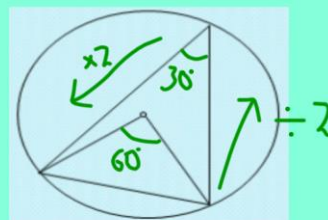
Mixed test questions:
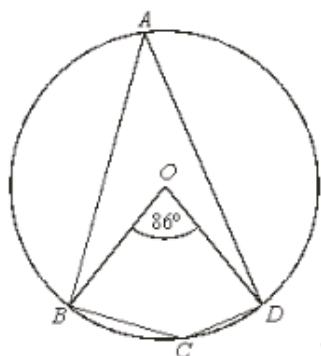
Diagram **NOT** accurately drawn

A, B, C and D are points on the circle, centre O.
Angle $BOD = 86°$

(a) (i) Work out the size of angle BAD.

.... °

(ii) Give a reason for your answer.

....

....

(2)
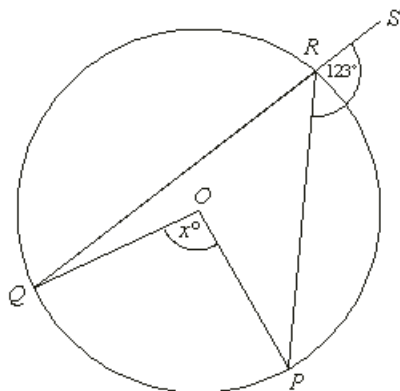
Diagram **NOT**
accurately drawn

QRS is a straight line.
QR and PR are chords of a circle, centre O.
Angle $PRS = 123°$.
Angle $QOP = x°$.

Calculate the size of the angle marked $x°$.
Give reasons for your answer.
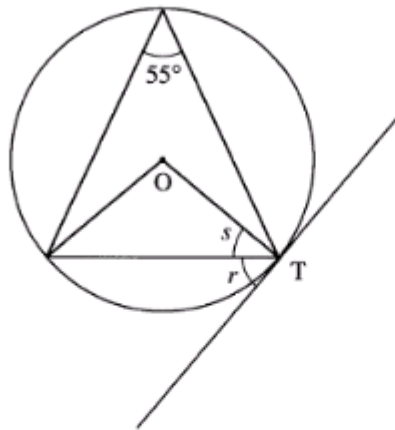
Not to scale

O is the centre of the circle.
T is the point of contact of a tangent to the circle.

Work out angles *r* and *s*.
Give a reason for each answer.

r = ...° because ....

....

s = ...° because ....

.... [3]
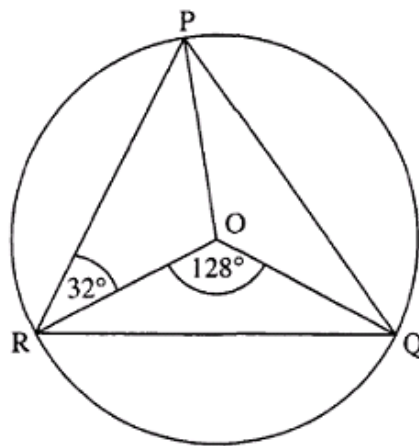
Not to scale

O is the centre of the circle.
Angle ROQ = 128°.
Angle PRO = 32°.

Prove that PQR is an isosceles triangle.
Give reasons for each step in your argument.

Quadratic Equation: key points

Try to factorise $x^2 + 5x + 2 = 0$
                        $(\quad)(\quad) = 0$

The Quadratic Formula will help us to find the two values for x that makes the equation equal 0.(roots)
Here is the quadratic formula known as 'The Formula'

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Try to factorise $x^2 + 5x + 2 = 0$

$a = 1$ ↗ $(\quad)(\quad) = 0$

                             $b = 5$ $c = 2$

Now substitute into the formula:

$$x = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$x = \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$x = \frac{-5 + \sqrt{17}}{2} \quad \text{or}$$

$$x = \frac{-5 - \sqrt{17}}{2}$$

Now put into a calculator to work out, you will have two answers:

So x = _____   or   x = _____

Some to try: remember to take the '-' sign if the number is negative!

5 Questions

1) $x^2+4x+1 = 0$

   $x=$ [____]   or $x=$ [____]

2) $x^2-3x+1 = 0$

   $x=$ [____]   or $x=$ [____]

3) $2x^2+x-8 = 0$

   $x=$ [____]   or $x=$ [____]

4) $3x^2-4x-9 = 0$

   $x=$ [____]   or $x=$ [____]

5) $3x^2-3x-2 = 0$
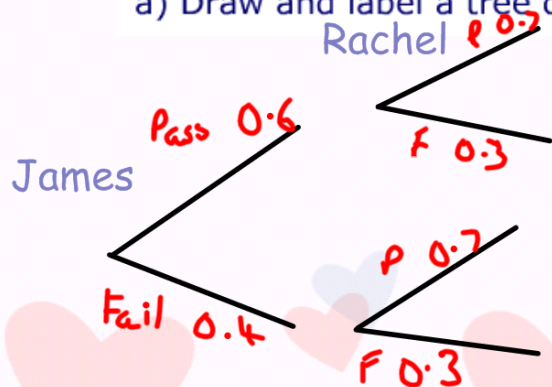
   $x=$ [____]   or $x=$ [____]

Probability

Key points:

Remember to multiply probabilities (do NOT add)

> Twins James and Rachel are taking their driving test next week. The probability that James will pass is 0.6; the probability that Rachel will pass is 0.7.
>
> These events are independent.
>
> a) Draw and label a tree diagram to show this information.

Q. What is the probability that they both pass?

James to Pass is 0.6   Rachel to pass is 0.7

Multiply them together so 0.6 x 0.7 =0.42

Top tip if you can't multiply decimals convert them to fractions:

Q. What is the probability only one of them passes?

    James to pass 0.6 Rachel to fail 0.3   0.6x0.3=0.18
    James to fail 0.4 Rachel to pass 0.7   0.4 x0.7=0.28

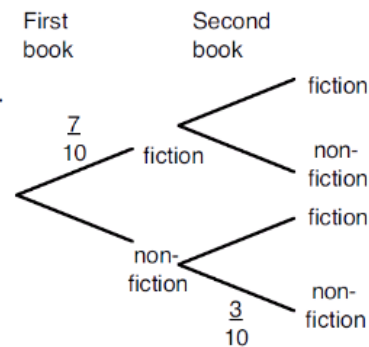   Now add your two answers together so 0.18+0.28= <u>0.46</u>

Each morning Bob and Bill catch the same bus. The probability that Bob catches the bus is 0.9 and for Bill it is 0.7. The probabilities are independent of each other.

a). Copy and **complete** the tree diagram.

b). Calculate the probability that on a given day :-

    i). they both catch the bus,

    ii). Bob catches the bus, but not Bill,

    iii). neither catch the bus,

    iv). at least one of them catch the bus.

Bob	Bill	
	0.7	catch
0.9 catch		not catch
		catch
not catch		not catch
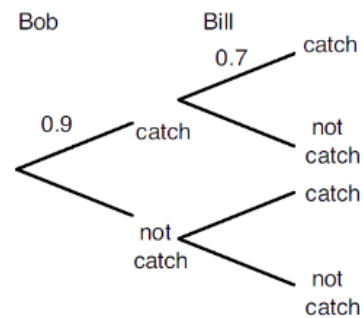
There are 10 books on a shelf in a library. Seven are fiction and three are nonfiction. A member of the public takes a book at random, looks at it, and then replaces it on the shelf. Another member of the public then takes a book at random from the shelf.

a). Copy and **complete** the tree diagram.

b). What is the probability the two books taken are :-

    i). both nonfiction,

    ii). both fiction,

    iii). one of each ?

First book	Second book	
		fiction
$\frac{7}{10}$ fiction		non-fiction
		fiction
non-fiction $\frac{3}{10}$		non-fiction

Extension

In a flower contest the probability that a red rose will win is 0.18, the probability that a yellow rose will win is 0.24. What is the probability that a red **or** yellow rose will win ?

<u>Sine and Cosine rules</u>

These are used only in non-right angled triangles. The formulae will be on your test paper at the front.

Two rules: Different applications

$a^2 = b^2 + c^2 - 2bc\text{Cos }A$ Cosine Rule

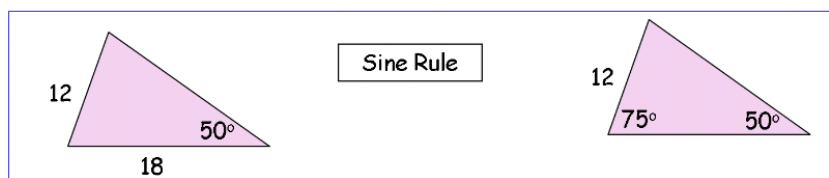$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$ Sine rule

Don't Panic, they will be on your formula Sheet!

When to use them:

The **Sine Rule** is used for cases in which the Cosine Rule cannot be applied. It is used to find:

1. An unknown side, when we are given two angles and a side.

2. An unknown angle when we are given two sides and an angle that is **not** included.
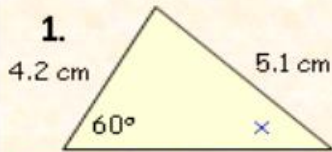

Sine Rule

The Cosine Rule

The Cosine rule can be used to find:

1. An unknown side when two sides of the triangle and the included angle are given.

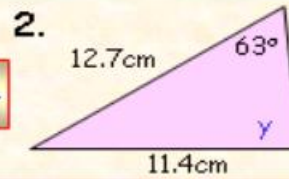2. An unknown angle when 3 sides are given.
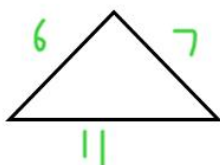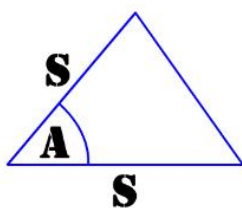
The Sine Rule

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

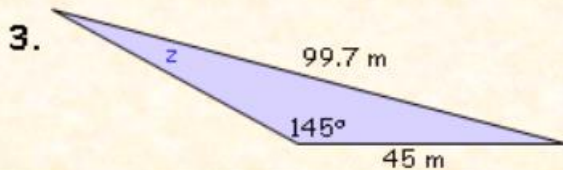To find an unknown angle we need 2 sides and an angle not included.

1.

4.2 cm / 5.1 cm / 60° / x

$$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$$

$$\frac{Sinx}{4.2} = \frac{Sin60°}{5.1}$$

$$\Rightarrow Sinx = \frac{4.2\,Sin60°}{5.1}$$

$$\Rightarrow \hat{x} = 45.5°(1\ dp)$$

2.

12.7cm / 63° / y / 11.4cm — Not to scale

$$\frac{Siny}{12.7} = \frac{Sin63°}{11.4}$$

$$\Rightarrow Siny = \frac{12.7\,Sin63°}{11.4}$$

$$\Rightarrow \hat{y} = 83.0°(1\ dp)$$

3.

z / 99.7 m / 145° / 45 m

$$\frac{Sinz}{45} = \frac{Sin145°}{99.7}$$

$$\Rightarrow Sinz = \frac{45\,Sin145°}{99.7}$$

$$\Rightarrow \hat{z} = 15°(1\ dp)$$

The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\,CosA$$

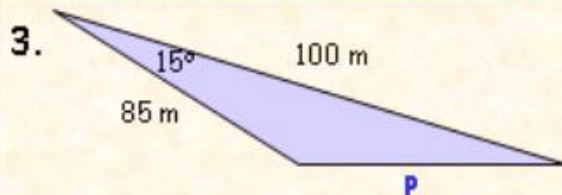To find an unknown side we need 2 sides and the included angle.
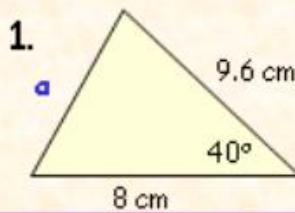
1.

a / 9.6 cm / 40° / 8 cm — Not to scale

$$a^2 = 8^2 + 9.6^2 - 2 \times 8 \times 9.6 \times Cos\ 40°$$

$$a = \sqrt{(8^2 + 9.6^2 - 2 \times 8 \times 9.6 \times Cos\ 40°)}$$

$$a = 6.2\,cm\ (1\ dp)$$

2.

7.7 cm / 65° / 5.4 cm / m

$$m^2 = 5.4^2 + 7.7^2 - 2 \times 5.4 \times 7.7 \times Cos\ 65°$$

$$m = \sqrt{(5.4^2 + 7.7^2 - 2 \times 5.4 \times 7.7 \times Cos\ 65°)}$$

$$m = 7.3\,cm\ (1\ dp)$$

3.

15° / 100 m / 85 m / P
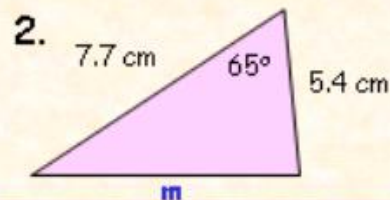
$$p^2 = 85^2 + 100^2 - 2 \times 85 \times 100 \times Cos\ 15°$$

$$p = \sqrt{(85^2 + 100^2 - 2 \times 85 \times 100 \times Cos\ 15°)}$$

$$p = 28.4\,m\ (1\ dp)$$

Tip: It does not matter how you label the sides as long as a side and angle are opposite each other e.g.

1.
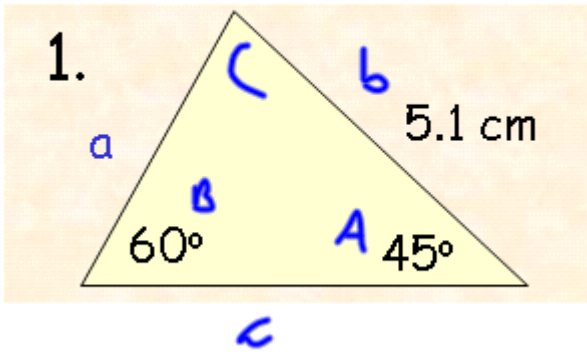
Mixed questions. Find which rule to use first. Find all missing measurements/ angles.

1).

17 m, 12 m, 24 m — D, E, F

2).

K, 8.7 cm, 76°, 6.4 cm — M, L

3).

E, 72°, 25 m, 47 m — D, F

4).

E, 78°, 43°, G, 29 m, F

5).

V, 19 mm, 103°, W, 28 mm, U

6).

N, 1.2 cm, 2.1 cm, 3.2 cm — M, L

7).

N, 148 m, L, 109°, 163 m, M

8).

Z, 6.7 m, Y, 45°, 8.9 m, W

9).

G, 8.6 cm, 20°, 126°, H, F

10).

Y, 37 mm, W, 121°, 56 mm

11).

N, 4 cm, 5 cm, P, 7 cm, O

12).

E, 34 m, 25 m, D, 47 m, F

Frequency Density $= \dfrac{\text{Frequency}}{\text{Classwidth}}$

Just remember that: Frequency = Area

Here is our table.
Complete the frequency densities.

Mileage m	Freq	Width	Height Freq Den
240 ⩽ m < 280	3	40	0.075
280 ⩽ m < 320	6		
320 ⩽ m < 340	8		
340 ⩽ m < 360	11		
360 ⩽ m < 380	6		
380 ⩽ m < 420	4		
420 ⩽ m < 460	2		

The first one has been done for you.

Area = Width × Height

So

Height (frequency density) = Area ÷ Width (freq)

= 3 ÷ 40 (freq)

= 0.075
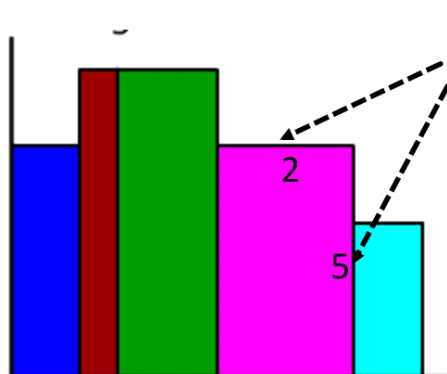
We multiply the length of the bar by the width of the bar to find out the frequency of a histogram
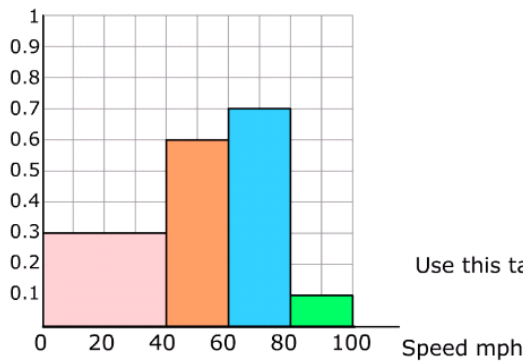
2

5

Questions to try

Type 1

The histogram shows the speeds, in miles per hour, of vehicles passing a
police check point.
Calculate an estimate of the mean speed of these 40 vehicles.

f.d.

Speed	Frequency
0 < v ≤ 40	
40 < v ≤ 60	
60 < v ≤ 80	
80 < v ≤ 100	

Markit

Use this table to help you find the answer.

Type 2: Remember to calculate frequency density!

One Monday, Victoria measured the time, in seconds, that individual birds spent on her
bird table.

She used this information to complete the frequency table.

Time (t seconds)	Frequency
0 < t ☐ 10	8
10 < t ☐ 20	16
20 < t ☐ 25	15
25 < t ☐ 30	12
30 < t ☐ 50	6

(a) Use the table to complete the histogram.

Frequency
density

Time (seconds)